Section 1.5 Parametric Relations and Inverses

A natural way to define functions is to define both elements of the ordered pair (x,y) in terms of another variable t (called a parameter).

Example 1) Consider the set of all ordered pairs (x,y) defined by the equations

$x = t + 1$

$y = t^2 + 2t$ **where t is any real number.**

a) Find the points determined by t= -3,-2,-1,0,1,2,3

b) Find an algebraic relationship between x and y (this will eliminate the parameter). Is y a function of x?

c) Graph the relation (by hand) in the (x,y) plane.

Inverse Relations and Inverse Functions

The ordered pair (a,b) is in a relation if and only if the ordered pair (b,a) is in the inverse relation.

Will the inverse of a function also be a function?

For a graph, the HORIZONTAL LINE TEST can help determine whether or not the inverse is also a function. The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

One-to-one function: Each x is paired with exactly one y and each y is paired with exactly one x.

If *f* is one-to-one with domain D and range R then the inverse of *f*, f^{-1} is the function with domain R and range D defined by

$f^{-1}(b) = a$ **if and only if** $f(a) = b$

Examples

a) Find an equation for $f^{-1}(x)$ **if** $f(x)=\frac{x}{x+1}$

change f(x) to y and solve for the other variable x.

Inverse can be found by undoing.

b) Find an equation for $f^{-1}(x)$ **if** $f(x)=\sqrt{x+3}$

State any restrictions on the domain of *f* and f^{-1}

c) Show algebraically that the following functions are inverses by <u>the Inverse Composition Rule.</u>

$f(g(x)) = x$ **and** $g(f(x)) = x$

<u>Show every step neatly. You may only cancel one term per step.</u>

$f(x) = 3x - 2$ **and** $g(x) = \dfrac{x+2}{3}$